**(c)(i)** Prove that the set S= {0,1,2,3} forms a ring under addition and multiplication modulo 4 but not a field.

**SOL: Sol:**The composite table for the two operations is given by

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| +4 | 0 | 1 | 2 | 3 |
| 0 | 0 | 1 | 2 | 3 |
| 1 | 1 | 2 | 3 | 0 |
| 2 | 2 | 3 | 0 | 1 |
| 3 | 3 | 0 | 1 | 2 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ×4 | 0 | 1 | 2 | 3 |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 2 | 3 |
| 2 | 0 | 2 | 0 | 2 |
| 3 | 0 | 3 | 2 | 1 |

As we can see from the composite table, the set S is a ring since it is

1.Closure under addition. For all a, b in S, the result of the operation a + b is also in S.

2.Associativity of addition. For all a, b and c in R, the equation (a + b) + c = a + (b + c) holds.

3.Existence of additive identity.There exists an element 0 in S, such that for all elements a in S, the equation 0 + a = a + 0 = a holds.Here the additive identity is given by 0.

4.Existence of additive inverse. For each a in S, there exists an element b in S such that a + b = b + a = 0

5.Commutativity of addition.For all a, b in S, the equation a + b = b + a holds

6.Closure under multiplication.For all *a*, *b* in S, the result of the operation *a* **·** *b* is also in S.

7.Associativity of multiplication.For all *a*, *b*, and *c* in S, the equation (*a* **·** *b*) **·** *c* = *a* **·** (*b* **·** *c*) holds.

8.Existence of multiplicative identity.There exists an element *1* in S, such that for all elements *a* in S, the equation *1* **·** *a* = *a* **·** *1* = *a* holds. Here the multiplicative identity is given by 1.

9 Dstributive laws For all *a*, *b* and *c* in S, the equation *a* **·** (*b* + *c*) = (*a* **·** *b*) + (*a* **·** *c*) holds.

For all *a*, *b* and *c* in S, the equation (*a* + *b*) **·** *c* = (*a* **·** *c*) + (*b* **·** *c*) holds

**Hence the set S= {0,1,2,3} forms a ring under addition and multiplication modulo 4**

Now we check (S , +4, ×4 )for field:

(S , +4, ×4 ) is a ring .So S can be a field if it

(1) is commutative (2) has multiplicative identity element(3) is such that every non-zero element has multiplicative inverse in R.

Here condition (1) and (2) are satisfied but

**Q2**. Consider the group G={1,2,3,4,5} under multiplication modulo 7

1. Find the multiplication table of G
2. Find 2-1, 3-1, and 6-1
3. Find order and subgroups generated by 2 and 3
4. Is G is cyclic?

**Sol 2:**

1. To find the a\*b in G, find the remainder when the product ab is divided by 7. For example, 5.6=30 which yields a remainder of 2 when divided by 7, hence 5\*6=2 in G.
2. Note first that 1 is the identity element of G. recall that element of G such that aa-1=1

Hence 2-1=4, 3-1=5, and 6-1=6.

1. We have 21=2, 22=4, but 23=1. Hence mod(2)=3 and gp(2)={1,2,4}. We have 31=3, 32=2, 33=6, 34=4, 35=5, 36=1. Hence mod(3)=6 and gp(3) = G.
2. G is Cyclic since G= gp(3).

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **\*7** | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 1 | 2 | 3 | 4 | 5 | 6 |
| 2 | 2 | 4 | 6 | 1 | 3 | 5 |
| 3 | 3 | 6 | 2 | 5 | 1 | 4 |
| 4 | 4 | 1 | 5 | 2 | 6 | 3 |
| 5 | 5 | 3 | 1 | 6 | 4 | 2 |
| 6 | 6 | 5 | 4 | 3 | 2 | 1 |